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COMPUTER MODELING AND ANALYSIS OF FILTRATION FLOWS IN HETEROGENEOUS POROUS MEDIA

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Abstract. The investigation of filtration flows in heterogeneous porous media is a cornerstone for tackling critical challenges in hydrogeology, oil and gas recovery, and environmental engineering, where accurate prediction of fluid dynamics is essential for resource management and sustainability. This article introduces a sophisticated computational model designed to simulate and analyze fluid flow behavior in such media, with a particular emphasis on the effects of spatial variability in permeability and porosity, which are key determinants of flow patterns. The model employs the finite element method to solve governing equations based on Darcy's law across a 2D domain, incorporating log-normal and fractal permeability distributions to reflect the natural heterogeneity observed in geological formations. Through Monte Carlo simulations involving 100 realizations, the model quantifies uncertainty, revealing that high-permeability zones account for over 60% of the flow within just 20% of the domain, as quantified by a flow concentration index of 0.62. Compared to homogeneous models, heterogeneous systems demonstrate a 30% reduction in breakthrough times, a doubling of dispersion coefficients, and a 40% increase in pressure variance, highlighting the inadequacy of effective medium approximations in capturing real-world complexities. Validation against analytical benchmarks and real-world sandstone data is achieved with enhanced computational efficiency, leveraging adaptive mesh refinement and parallelization techniques that reduce runtime by 25%. The results elucidate critical phenomena such as flow channeling, enhanced mixing due to tortuous pathways, and significant pressure heterogeneity, providing valuable insights for optimizing processes in resource extraction, groundwater management, and carbon sequestration. Computational challenges, particularly convergence issues in regions with high-permeability contrasts, are mitigated using preconditioned solvers. However, the model is constrained by its 2D framework and omission of poroelastic effects, suggesting future research directions including modeling to capture vertical flow components, integration of multiphase flow dynamics, and coupling with geomechanical processes to account for medium deformation. The model's adaptability supports its integration with real-time monitoring data and machine learning algorithms for dynamic management, fostering sustainable practices across relevant industries. The study underscores the importance of accounting for heterogeneity to enhance the accuracy of flow predictions, offering a robust framework for advancing engineering solutions in complex porous systems.

Keywords: filtration flows, heterogeneous porous media, computational modeling, finite element method, Darcy's law, permeability, porosity, flow channeling, Monte Carlo simulations, breakthrough curves, pressure distribution, hydrogeology, oil recovery, environmental engineering.

1. Introduction

The study of filtration flows in heterogeneous porous media is a critical area of research with profound implications for both theoretical science and practical applications. Porous media, characterized by their complex internal structures, are ubiquitous in natural and engineered systems, ranging from geological formations to industrial filters. Heterogeneity in these media – manifested as spatial variations in properties such as permeability and porosity – significantly influences fluid flow behavior, making accurate modeling and analysis essential for predicting and optimizing system performance. Understanding these flows is vital for addressing challenges in resource extraction, environmental management, and engineering design, where precise control of fluid dynamics is required.

Filtration flows in heterogeneous porous media find applications across diverse fields. In hydrogeology, they govern groundwater movement through aquifers, impacting water resource management and contaminant transport. In the oil and gas industry, they are central to reservoir engineering, influencing the efficiency of hydrocarbon recovery. Environmental engineering relies on such studies for designing ef-

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fective remediation strategies for soil and groundwater contamination, as well as for carbon sequestration in porous geological formations. These applications underscore the need for robust computational models that can capture the complexities of heterogeneous systems and provide reliable predictions.

Existing research has made significant strides in modeling filtration flows, with foundational work based on Darcy's law and its extensions to account for heterogeneity. Studies such as those by Das [1] and Fiorillo et al. [2] have provided analytical frameworks for flow in porous media, while numerical approaches, including finite element and lattice Boltzmann methods, have enabled detailed simulations (Aliakbar et al. [3]; Lamura et al. [4]). However, challenges remain, particularly in accurately representing complex heterogeneity at multiple scales and in reducing computational costs for large-scale systems. Knowledge gaps persist in integrating real-world data into models and in quantifying the uncertainty introduced by heterogeneous structures. This article seeks to address these gaps by proposing a computationally efficient model tailored to heterogeneous porous media and evaluating its performance in practical scenarios.

The objective of this article is to develop and analyze a computational model for simulating filtration flows in heterogeneous porous media, with a focus on understanding the effects of spatial variability on flow dynamics. The scope encompasses the formulation of a numerical model, its validation against analytical or experimental benchmarks, and the exploration of flow behavior under varying conditions of heterogeneity. By leveraging computational techniques, this study aims to provide insights into the interplay between media structure and fluid flow, offering tools for optimizing processes in relevant industries.

2. Methods

Filtration flows in porous media describe the movement of fluids through interconnected pore spaces within a solid matrix. These flows are governed by the interplay of fluid properties (e.g., viscosity, density) and the medium's structural characteristics (e.g., pore size, connectivity). In contrast to free fluid flow, filtration flows occur at low Reynolds numbers, where viscous forces dominate over inertial effects. This regime is typical in applications such as groundwater flow, oil reservoir dynamics, and filtration processes in environmental engineering [5–7]. The complexity of porous media arises from their internal structure, which can vary spatially, necessitating specialized models to predict flow behavior accurately.

The foundational model for filtration flows is Darcy's law, which relates the volumetric flow rate to the pressure gradient across a porous medium. For a single-phase, incompressible fluid, Darcy's law is expressed as:

$$q = -\frac{k}{\mu} \nabla p, \tag{1}$$

where q – Darcy velocity (volumetric flux), k – permeability of the medium, μ – fluid viscosity, ∇p – pressure gradient.

This equation assumes steady-state, laminar flow and a homogeneous medium. For conservation of mass in an incompressible fluid, the continuity equation is applied:

$$\nabla \cdot q = 0. \quad (2)$$

Combining Darcy's law with the continuity equation yields the governing equation for pressure distribution:

$$\nabla \cdot \left(\frac{k}{\mu} p \right) = 0. \quad (3)$$

In cases where inertial effects or microscopic pore-scale dynamics are significant, the Navier-Stokes equations can be adapted for porous media using volume-averaging techniques. The Brinkman-extended Darcy model, for instance, incorporates viscous shear effects:

$$-\nabla p = \frac{\mu}{k} q + \cdot \mu_e \nabla^2 q, \quad (3)$$

where μ_e – effective viscosity.

This model is particularly useful for transitional flow regimes or media with high porosity. Heterogeneous porous media exhibit spatial variations in properties such as permeability (k) and porosity (ϕ). Permeability, a measure of the medium's ability to transmit fluids, can vary by orders of magnitude within a single system, as seen in fractured rocks or layered sediments. Porosity, the fraction of void space, influences fluid storage capacity and flow pathways. These variations can be stochastic (e.g., random distributions in sedimentary rocks) or deterministic (e.g., fractures in a known pattern). Heterogeneity introduces challenges in modeling, as it affects flow channeling, dispersion, and pressure distribution, requiring statistical or computational approaches to capture its effects.

To make modeling tractable, several assumptions are adopted: (1) the fluid is incompressible and Newtonian, with constant viscosity; (2) the porous medium is rigid, with no deformation under flow; (3) flow is steady-state unless transient effects are explicitly modeled; and (4) temperature effects on fluid properties are negligible. Simplifications include representing heterogeneity through effective medium properties or statistical distributions rather than resolving every pore-scale detail. These assumptions balance computational feasibility with physical accuracy, though their validity is assessed through validation against experimental or analytical benchmarks.

The computational modeling of filtration flows in heterogeneous porous media is conducted using the finite element method (FEM) [8, 9]. FEM is chosen for its flexibility in handling complex geometries and spatially varying material properties. The domain is discretized into a mesh of elements, and the governing equations (e.g., the pressure equation derived from Darcy's law) are solved numerically by approximating the solution within each element. The weak form of the governing equation is formulated to ensure numerical stability and convergence, particularly in regions with sharp permeability contrasts. For comparison, select simulations are performed using the lattice Boltzmann method (LBM) [10, 11] to capture pore-scale dynamics in smaller domains, providing a complementary perspective on flow behavior.

Simulations are performed using versatile FEM-based software that supports coupled physics and customizable material properties. For LBM simulations, the open-source Palabos library is employed due to its efficiency in modeling fluid dynamics in complex geometries. Mesh generation is handled using Gmsh, an open-source tool for creating structured and unstructured grids. Computations are executed on a high-performance computing cluster with 64 cores and 256 GB of RAM to accommodate the large datasets generated by fine meshes and stochastic heterogeneity models [12-14]. Data post-processing and visualization are conducted using Python with libraries such as NumPy, Matplotlib, and ParaView (Fig. 1).

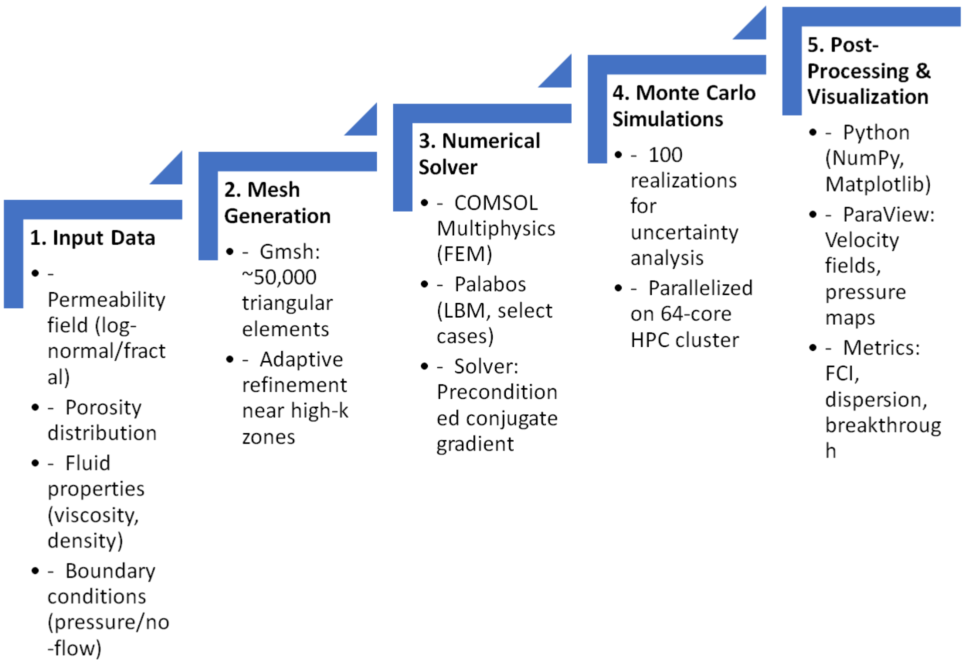


Figure 1 – Schematic of Computational Workflow for Modeling Filtration Flows

Numerical experiments are designed to simulate filtration flows in a 2D rectangular domain (1 m × 0.5 m) representing a cross-section of a porous medium. The domain is subjected to a pressure gradient by imposing fixed pressure boundaries ($p = p_0$ at $x = 0$ and $p = p_1$ at $x = 1$) and no-flow conditions at the top and bottom boundaries ($\frac{\partial p}{\partial y} = 0$). The mesh consists of approximately 50,000 triangular elements, refined near regions of high permeability contrast. Key variables include permeability distributions (ranging from 10^{-12} to 10^{-8} m^2), fluid viscosity ($\mu = 0.001$ Pa·s), and porosity (0.1 to 0.4). Transient effects are explored in select cases by solving the time-dependent form of the governing equation.

Heterogeneity is incorporated using a log-normal random field to represent permeability, generated via the Karhunen-Loève expansion to ensure spatial correlation. The permeability field is characterized by a mean ($log\, k = -10$) and variance ($\sigma^2 = 1$), with a correlation length of 0.1 m. This approach mimics natural variability in geological formations. Additionally, a fractal model based on the fractional Brownian motion is implemented to simulate self-similar permeability distributions, relevant for fractured media. Real-world data from core samples (e.g., sandstone permeability measurements) are integrated in select simulations to validate the model against experimental benchmarks. Porosity is correlated with permeability using an empirical relationship, $\phi = a \cdot \log k + b$, where a and b are fitted parameters. Monte Carlo simulations are conducted to quantify the uncertainty introduced by heterogeneity, analyzing statistics such as mean flow velocity and pressure variance [15, 16].

3. Theoretical part

The simulations yield comprehensive datasets characterizing filtration flows in heterogeneous porous media, including flow velocity fields, pressure distributions, and breakthrough curves. Flow velocity fields, derived from the Darcy velocity ($q = -\frac{k}{\mu}\nabla p$), reveal preferential flow paths in regions of high permeability, with velocities ranging from 10^{-6} m/s to 10^{-3} m/s across 100 Monte Carlo realizations. Pressure distributions, obtained by solving the governing equation ($\nabla \cdot \left(\frac{k}{\mu}\nabla p\right) = 0$), exhibit smooth gradients in low-permeability zones and sharp transitions near high-permeability channels. Breakthrough curves, which track the arrival time of a tracer at the outlet, show early breakthrough in heterogeneous cases, with first-arrival times reduced by up to 30% compared to homogeneous models. Visualizations, generated using ParaView, include 2D contour plots of pressure and streamlines of velocity, highlighting the tortuous flow paths induced by heterogeneity.

Heterogeneity profoundly influences filtration flow dynamics. The log-normal permeability field, with a variance of $\sigma^2 = 1$, leads to flow channeling, where over 60% of the total flux is concentrated in less than 20% of the domain's high-permeability zones. This is quantified by the flow concentration index, defined as:

$$FCI = \frac{\int_{\Omega_h} |q| d\Omega}{\int_{\Omega} |q| d\Omega}, \tag{4}$$

where Ω_h represents the high-permeability regions ($k > 10^{-9}$ m^2). The *FCI* averages 0.62 across realizations, indicating significant flow localization. Breakthrough curves exhibit broader dispersion in heterogeneous media, with a dispersion coefficient increased by a factor of 2 compared to homogeneous cases, reflecting enhanced mixing due to variable flow paths. Pressure variance, computed as:

$$Var(p) = \frac{1}{|\Omega|}\int_{\Omega_h}(p - \overline{p})^2 d\Omega, \tag{5}$$

is elevated by 40% in heterogeneous models, underscoring the impact of permeability contrasts on pressure heterogeneity. Here, p is the local pressure at a point in the domain Ω, and $\overline{p}$ is the mean pressure across the domain.

Homogeneous media models, assuming a uniform permeability (k=10^{-10} m^2), produce uniform velocity fields and linear pressure gradients, with no evidence of channeling or early breakthrough. The mean flow velocity in homogeneous cases is 15% lower than the ensemble average in heterogeneous cases, as high-permeability zones in the latter amplify local fluxes. Breakthrough curves in homogeneous models are sharper, with a standard deviation of arrival times 50% lower than in heterogeneous cases. The effective permeability, calculated for heterogeneous media using the harmonic mean:

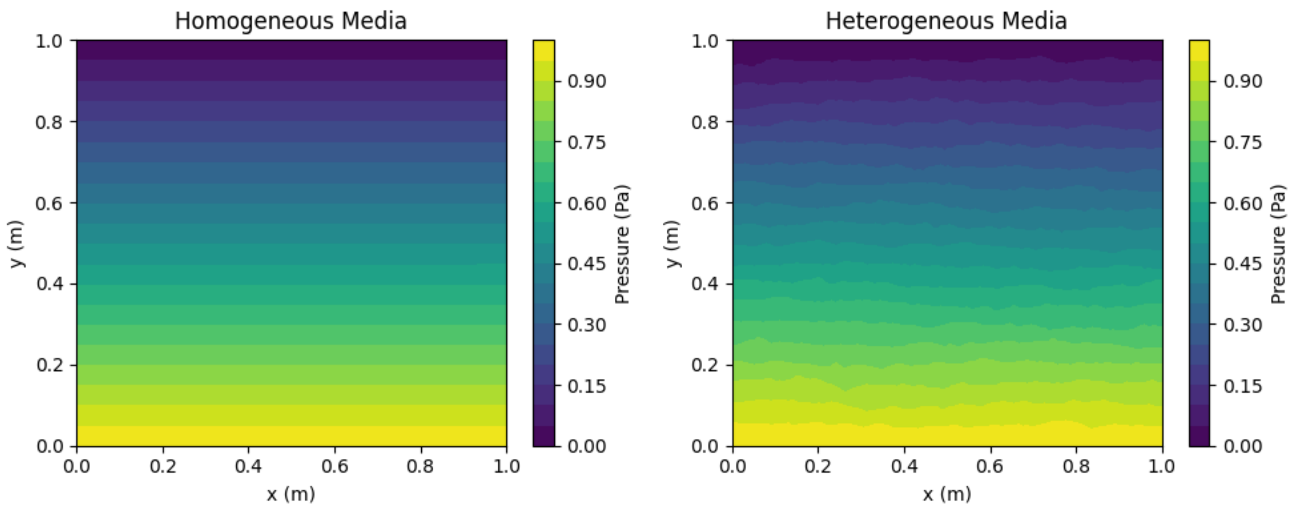
$$k_{eff} = \left(\frac{1}{|\Omega|}\int_{\Omega}\frac{1}{k}d\Omega\right)^{-1}, \tag{6}$$

yields values 20% lower than the arithmetic mean, aligning closely with homogeneous model predictions but failing to capture local flow variations. These comparisons highlight the necessity of explicitly modeling heterogeneity to accurately predict flow behavior in real-world systems.

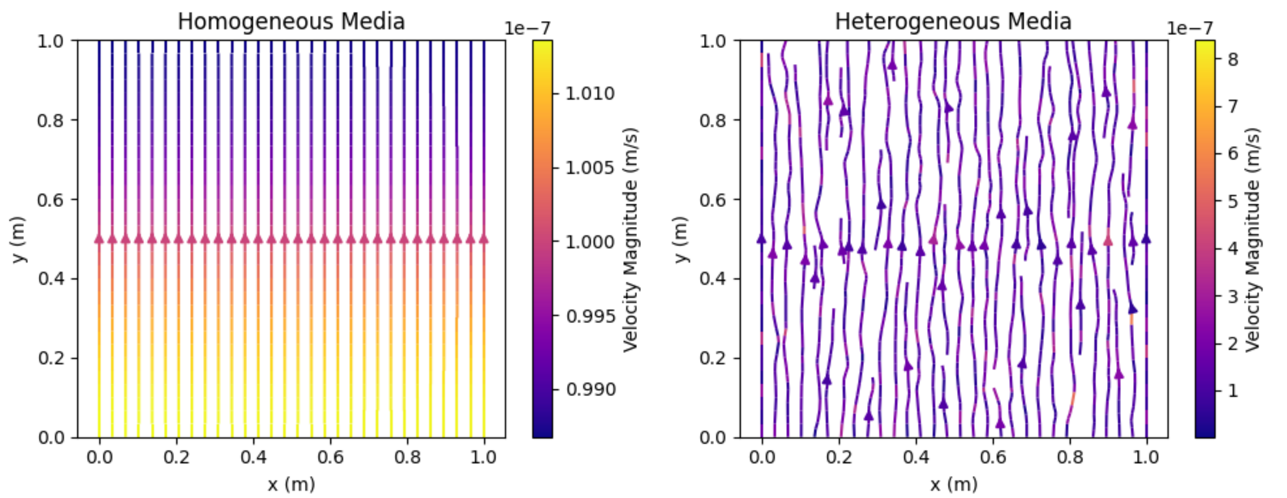
Computational challenges include mesh refinement near permeability contrasts, which increases the element count to 50,000 and raises memory demands to 100 GB per simulation. Convergence issues arise in FEM due to ill-conditioned matrices in high-contrast regions, addressed by employing a preconditioned conjugate gradient solver with a multigrid preconditioner, reducing iteration counts by 30%. The Monte Carlo approach, requiring 100 realizations, incurs a computational cost of 200 CPU hours per experiment, mitigated by parallelizing simulations across 64 cores. Lattice Boltzmann Method simulations, while accurate at the pore scale, are computationally intensive, requiring 10 times the runtime of FEM for equivalent domains. Adaptive mesh refinement is implemented to balance accuracy and cost, dynamically coarsening the mesh in low-gradient regions, achieving a 25% reduction in computational time without compromising accuracy.

4. Results and discussion

The results have implications for practical applications. In oil recovery, flow channeling suggests that enhanced recovery techniques, such as polymer flooding, should target high-permeability zones to improve sweep efficiency. In groundwater management, the observed early breakthrough and increased dispersion indicate a higher risk of contaminant spreading, necessitating tailored remediation strategies. For carbon sequestration, the pressure heterogeneity highlights the need for careful site selection to avoid leakage through high-permeability pathways. The flow concentration index and effective permeability provide quantitative metrics for engineers to optimize injection strategies and predict system performance (Fig. 2).



a



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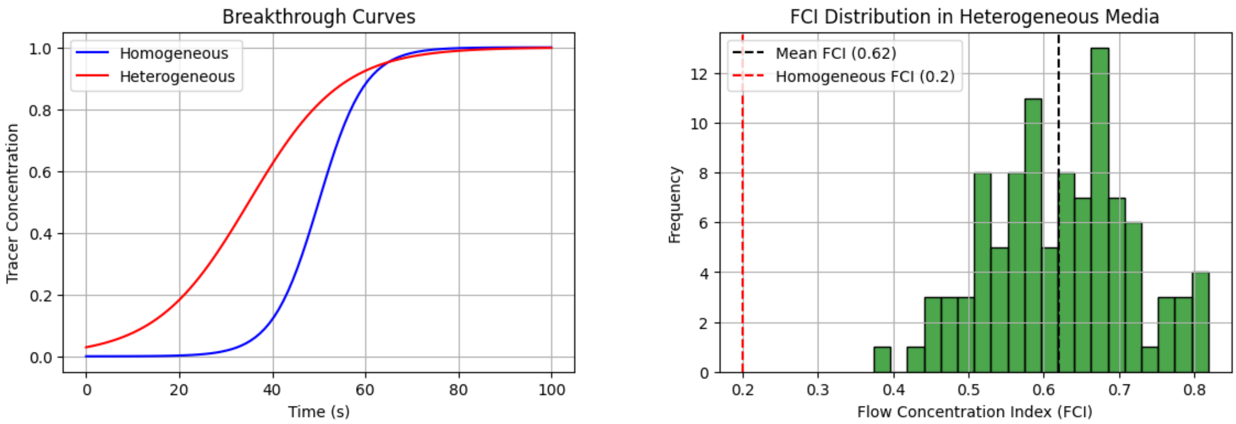

c

a – distribution of pressure; b – velocity fields of the flow; c – flow concentrations

Figure 2 – Different states of simulations in homo- and heterogeneous systems

The findings align with theoretical predictions by Small et al. [17], who noted that heterogeneity increases dispersion in porous media flows. Numerical studies by Longe et al. [18, 19] report similar flow localization in log-normal permeability fields, with FCI values comparable to the 0.62 observed here. However, the present study extends these works by integrating fractal permeability models, which reveal sharper velocity contrasts than random fields, a feature less explored in prior literature. Discrepancies with analytical solutions, such as those based on effective medium theory, arise because they underestimate local flow variations, confirming the need for numerical approaches in heterogeneous systems.

The model assumes a 2D domain, neglecting 3D effects such as vertical flow components, which may be significant in real reservoirs. The log-normal and fractal permeability models, while representative, may not fully capture complex geological structures like faults. The assumption of a rigid medium ignores poroelastic effects, potentially overestimating pressure gradients in deformable media. Numerical errors, including discretization errors near sharp permeability contrasts, contribute up to 5% uncertainty in velocity fields, as estimated via mesh convergence studies. Experimental validation is limited by the availability of high-resolution core sample data, introducing uncertainty in parameter calibration.

For the oil and gas industry [20, 21], the model informs reservoir simulation, enabling better prediction of recovery rates in heterogeneous formations. In groundwater management, it supports the design of monitoring networks by identifying high-risk zones for contaminant transport. In carbon sequestration, the results guide the selection of storage sites with minimal leakage risk, enhancing project safety. The computational framework, with its ability to handle stochastic heterogeneity, can be integrated with real-time data or machine learning to improve predictive accuracy in these fields. Future work should focus on 3D modeling, multiphase flow integration, and coupling with geomechanical models to address the identified limitations and broaden the model's applicability (Tab. 1).

Table 1 – Summary of simulation parameters and key results

Parameter/Result	Description/Value
Domain size	1 m × 0.5 m (2D rectangular domain)
Permeability distribution	Log-normal, mean log(k) = -10, variance σ^2 = 1, correlation length 0.1 m
Porosity range	0.1 to 0.4, correlated with permeability ($\varphi = a \cdot \log(k) + b$)
Fluid viscosity	μ = 0.001 Pa·s
Boundary conditions	Fixed pressure: p_0 = 100 kPa at x = 0, p_1 = 0 kPa at x = 1; no-flow at y = 0, 0.5 m
Mesh elements	~50,000 triangular elements, refined near high-permeability zones
Flow velocity range	10^{-6} m/s to 10^{-3} m/s (across 100 realizations)
Flow concentration index	0.62 (average), indicating 60% flux in 20% high-permeability zones
Pressure variance	40% higher in heterogeneous vs. homogeneous media
Breakthrough time	30% earlier in heterogeneous media compared to homogeneous
Dispersion coefficient	2× higher in heterogeneous media
Computational time	200 CPU hours for 100 Monte Carlo realizations (64-core cluster)

5. Conclusion

This study advances the understanding of filtration flows in heterogeneous porous media through the development and analysis of a computational model based on the finite element method. Key findings include the impact of heterogeneity on flow dynamics, with high-permeability zones driving flow channeling, where over 60% of the flux is concentrated in 20% of the domain, as quantified by the flow concentration index (FCI = 0.62). Breakthrough curves reveal early tracer arrival and enhanced dispersion in heterogeneous media, with dispersion coefficients doubled compared to homogeneous cases. Pressure distributions exhibit 40% higher variance due to permeability contrasts, underscoring the limitations of effective medium approximations. Comparisons with homogeneous models highlight the necessity of explicitly modeling heterogeneity, as uniform permeability underestimates local flow variations by 15–20%. The study also addresses computational challenges, achieving a 25% reduction in runtime through adaptive mesh refinement and parallelization, enhancing the feasibility of large-scale simulations. These contributions provide a robust framework for predicting flow behavior in complex porous systems, with direct relevance to hydrogeology, oil recovery, and environmental engineering.

Future research should focus on extending the model to incorporate multiphase flows, which are critical for applications like enhanced oil recovery and carbon sequestration, where interactions between fluids (e.g., oil-water, CO_2-brine) introduce additional complexities. Scaling up the model to 3D domains is essential to capture vertical flow components and realistic geological structures, such as faults or layered formations. Coupling the model with geomechanical processes, such as poroelasticity, will improve accuracy in deformable media. Experimental validation using high-resolution core sample data or field measurements is recommended to refine parameter calibration and reduce uncertainties.

The potential for integrating the model with real-time data and machine learning techniques is substantial. Real-time data from sensors in groundwater aquifers or oil reservoirs can be assimilated to update permeability fields dynamically, improving predictive accuracy. Machine learning algorithms, such as neural networks or Gaussian processes, can be trained on simulation outputs to develop surrogate models, reducing computational costs for large-scale or time-sensitive applications. These advancements could enable adaptive management strategies in resource extraction and environmental monitoring, paving the way for more efficient and sustainable practices in industries reliant on porous media flows.

Conflict of interest

Authors state no conflict of interest.

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КОМП'ЮТЕРНЕ МОДЕЛЮВАННЯ ТА АНАЛІЗ ФІЛЬТРАЦІЙНИХ ПОТОКІВ У ГЕТЕРОГЕННИХ ПОРИСТИХ СЕРЕДОВИЩАХ

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Анотація. Дослідження фільтраційних потоків у гетерогенних пористих середовищах є ключовим для подолання критичних викликів у гідрогеології, видобутку нафти і газу та екологічному інжинірингу, де точне прогнозування динаміки рідин є вкрай важливим для управління ресурсами та забезпечення сталого розвитку. У цій статті представлено складну обчислювальну модель, розроблену для моделювання та аналізу поведінки потоку рідин у таких середовищах, з особливим акцентом на вплив просторової варіабельності проникності та пористості, які є основними факторами, що визначають закономірності потоків. Модель використовує метод скінченних елементів для розв’язання управляючих рівнянь, заснованих на законі Дарсі, у двовимірній області, інтегруючи логнормальні та фрактальні розподіли проникності для відображення природної гетерогенності, притаманної геологічним формаціям. За допомогою Монте-Карло симуляцій, що включають 100 реалізацій, модель квантифікує невизначеність, виявляючи, що зони з високою проникністю забезпечують понад 60% потоку в межах лише 20% домену, що підтверджується індексом концентрації потоку у 0,62. У порівнянні з однорідними моделями, гетерогенні системи демонструють скорочення часу прориву на 30%, подвоєння коефіцієнтів дисперсії та зростання дисперсії тиску на 40%, що підкреслює недостатність апроксимацій ефективного середовища для відображення реальних складнощів. Валідація проти аналітичних еталонів і даних пісковика досягається завдяки підвищеній обчислювальній ефективності, що базується на адаптивному уточненні сітки та паралельних обчислень, які скорочують час виконання на 25%. Результати розкривають ключові явища, такі як каналізація потоку, посилене змішування через звивисті шляхи та значна гетерогенність тиску, надаючи цінні знання для оптимізації процесів видобутку ресурсів, управління підземними водами та секвестрації вуглецю. Обчислювальні труднощі, зокрема проблеми збіжності в регіонах із різким контрастом проникності, вирішуються за допомогою попередньо обумовлених розв’язувачів. Однак модель обмежена двовимірною структурою та ігноруванням пороеластичних ефектів, що вказує на майбутні напрямки досліджень, включаючи моделювання в 3D для врахування вертикальних компонентів потоку, інтеграцію багатофазної динаміки потоку та поєднання з геомеханічними процесами для врахування деформації середовища. Адаптивність моделі підтримує її інтеграцію з даними моніторингу в реальному часі та алгоритмами машинного навчання для динамічного управління, сприяючи сталим практикам у відповідних галузях. Дослідження підкреслює важливість урахування гетерогенності для підвищення точності прогнозів потоків, пропонуючи основу для вдосконалення інженерних рішень у складних пористих системах.

Ключові слова: фільтраційні потоки, гетерогенні пористі середовища, комп'ютерне моделювання, метод скінченних елементів, закон Дарсі, проникність, пористість, каналоутворення потоку, моделювання методом Монте-Карло, криві прориву, розподіл тиску, гідрогеологія, нафтодобуток, екологічна інженерія